A Computation of $L(2k+1;\chi_4)$

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The Riemann-zeta function is one of the functions that mathematicians have considered tremendously due to its applications. This function is closely related to the distribution of prime numbers, which is essential to the modern world information security. Further, the usage of this function is widespread in several fields other than mathematics, such as physics, probability theory, and applied statistics. Despite a vast contribution of the function, its definition is a simple-looking infinite series of the form,

$$\zeta(s) := \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots; \tag{1}$$

where s is any complex number whose real part is greater than 1. For any infinite series, one of the frequently asked questions is if there is a formula for its exact values. In fact, in the case of the Riemann-zeta function, it is well-known that its values can be expressed as

$$\zeta(2k) := \sum_{n=1}^{\infty} \frac{1}{n^{2k}} = \frac{(-1)^{k-1} 2^{2k-1} \pi^{2k}}{(2k)!} B_{2k}; \tag{2}$$

where k is any positive integer and $\{B_{2k}\}$